Engineering Notes

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Simple Marching-Vortex Model for Two-Dimensional Unsteady Aerodynamics

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Introduction

THE purpose of this Note is to describe a simple analytical model for an unsteady airfoil with which accurate values of the normal force, moment, and leading-edge suction may be obtained. As part of ongoing studies of oscillating-wing propulsion, this model was developed specifically for use with small computers using a sequence of repetitive and straightforward calculations.

This model is based on the "marching-vortex" concept, where motion begins from an impulsive start with the subsequent generation of a vortex wake, modeled by a sequence of discrete vortices shed at equal time intervals. Thus, for steady-state motion, the force and moment responses are asymptotically achieved. An application of such a method is given by Fairgrieve and DeLaurier[1] for two-dimensional airfoils undergoing general periodic motions, using both planar and deformable discrete-vortex wakes. However, the airfoil's bound vorticity was continuous in order to provide accurate calculations for leading-edge suction. This had the drawback of requiring fairly time-consuming chordwise integrations to find the normal force and pitching moment at each time step. Also, it offered no simple extension to finite-span wings.

By comparison, Wells and Queijo[2] postulated an unsteady-airfoil model consisting of a single bound vortex at the quarter-chord with the incident-flow boundary condition met at the three-quarter chord location. However, they chose to model the wake generated by an incremental angle of attack with a single shed vortex, starting $c/4$ aft of the trailing edge and changing in strength as it traveled downstream. Furthermore, in order to best match the Wagner indicial-lift solution (described by Garrick[3]), the shed vortex had to travel at half the freestream velocity.

For the present model, it was decided to combine the single bound-vortex representation of Ref. 2 with the marching-vortex wake of Ref. 1 and to determine parameters such as shed-vortex spacing by comparison with certain exact solutions.

A nearly identical two-dimensional model was independently derived by Hancock and Lam[4] for the attached flow portion of their studies of dynamic stall. Since propulsion was not a consideration, only lift and pitching moment were calculated, although the details regarding these calculations were not provided.

For this present work, it will be shown, by comparison with exact solutions, that this simple model is sufficiently accurate for its intended applications, such as oscillating-airfoil propulsion.

Method of Analysis

The analytical model for an airfoil is shown in Fig. 1, where the chordwise bound vorticity is concentrated into a single bound vortex Γ_j at the $c/4$ location. The vortex wake is represented by a sequence of discrete vortices Γ'_i traveling downstream at velocity U. These vortices are shed at equal time intervals Δt evenly spaced in the wake by a distance $\delta c = U\Delta t$. The motion of the airfoil is described by the mid-chord downward plunging displacement h and the geometric pitch angle θ, measured relative to U.

At each time step, a new shed vortex Γ'_j appears a distance $\tilde{d}c$ aft of Γ_j with a corresponding change in the bound vortex strength according to the Helmholtz vorticity conservation law. A flow tangency condition at the $3c/4$ control point is used to determine the strength of Γ'_j. That is, the vortex-induced velocity normal to the chord must cancel the incident normal velocity from the airfoil's motion $U\alpha_j$. Since all previous wake vortex strengths are known from previous steps, a simple expression for Γ'_j is obtained by assuming that the vortex wake remains coplanar with the airfoil.

Further, when using this model to calculate the leading-edge suction T_{sj} (described later), it was found that the indicial starting value from Ref. 3 was matched only if $\tilde{d} = 1$. Therefore, the Γ'_j vortex appears one chord aft of Γ_j, the same arrangement used in Ref. 4. The resulting expression for Γ'_j is

$$\Gamma'_j = -\frac{1}{2}\left[\pi c U \alpha_j + \sum_{i=1}^{j-1}\left(\frac{1+(j-i)\delta}{(1/2)+(j-i)\delta}\right)\Gamma'_i\right] \tag{1}$$

The airfoil's normal force is comprised of circulatory and apparent-mass terms

$$N_j = N_{cj} + N_{aj} \tag{2}$$

The circulatory term N_{cj} is obtained from the vortex-impulse method described by Milne-Thomson[5]

$$N_{cj} = \frac{dI_j}{dt} \tag{3}$$

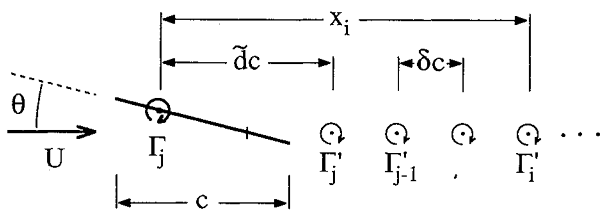

Fig. 1 Marching-vortex model.

Received July 31, 1989.

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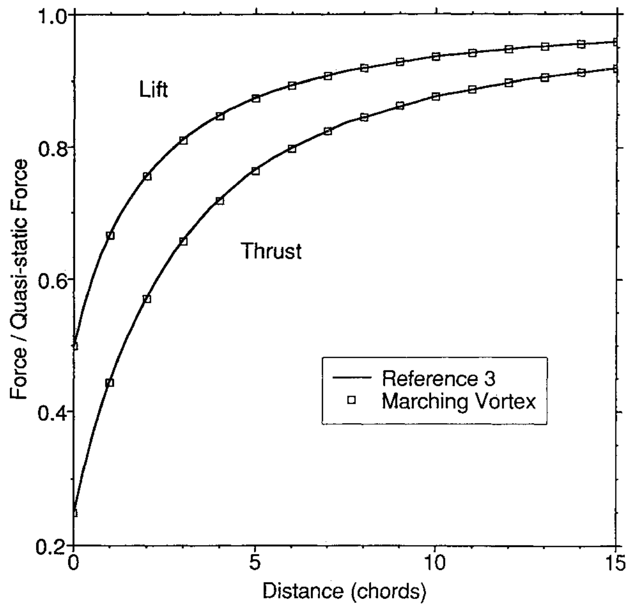

Fig. 2 Airfoil lift and thrust for indicial motion.

where I_j is the vortex impulse at instance j, which is

$$I_j = -\rho \sum_{i=1}^{j} \Gamma_i' x_i \tag{4}$$

where x_i is the distance between the bound vortex and the shed vortex Γ_i'. For this model, where I is not generally a continuous function, the derivative is evaluated at the instant the boundary condition is satisfied. This gives the result

$$N_{cj} = -\rho U \sum_{i=1}^{j} \Gamma_i' = \rho U \Gamma_j \tag{5}$$

The apparent-mass effect, as given by Garrick,[6] is

$$N_{aj} = 0.25 \rho \pi c^2 (\ddot{h}_j + U\dot{\theta}_j) \tag{6}$$

The expression for the leading-edge suction force T_{sj} was derived from Garrick's equation for an airfoil with continuous chordwise vorticity[6]:

$$T_{sj} = \rho \pi c (w_j - 0.25 c \dot{\theta}_j)^2 \tag{7}$$

where w_j, for this analysis, is the normal velocity at the $3c/4$ location due to the airfoil's motion and the vortex wake. So,

$$w_j = U\alpha_j + \frac{1}{2\pi c} \sum_{i=1}^{j} \frac{\Gamma_i'}{(1/2) + (j-i)\delta} \tag{8}$$

Finally, the midchord pitching moment is given by

$$M_j = 0.25 c N_{cj} + M_{aj} + M_{\theta j} \tag{9}$$

where M_{aj}, the apparent-mass contribution, and $M_{\theta j}$, due to pitch rate, are obtained from Ref. 6

$$M_{aj} = -(1/128) \rho \pi c^4 \ddot{\theta}_j \tag{10}$$

$$M_{\theta j} = -(1/16) \rho \pi c^3 U \dot{\theta}_j \tag{11}$$

Note that if the airfoil has a camber line approaching a circular arc, its steady-state normal force at zero angle of attack is simply added to Eq. (2).

Comparisons to Exact Solutions

The preceding equations were first compared to the Wagner solution for an indicially started wing as given by Garrick in

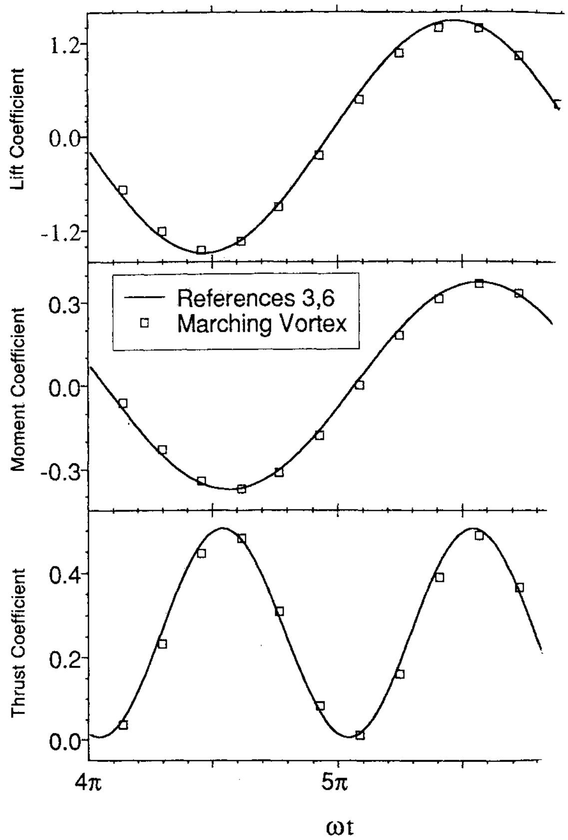

Fig. 3 Airfoil lift, thrust, and pitching moment for pitching and plunging motion, $k = 0.5$, $\delta = 0.5$.

Ref. 3. As mentioned, the distance from the bound vortex to the latest shed vortex $\tilde{d}$ was set equal to one in order to match the starting value for leading-edge suction. The resulting time-dependent marching-vortex values of lift and thrust, relative to the quasistatic results, are plotted in Fig. 2 along with the results from Ref. 3. These were obtained with a vortex spacing of $\delta = \tilde{d} = 1$ with excellent results for this case. Results for other vortex spacings differed by a few percent.

For airfoils undergoing oscillatory midchord plunging and pitching, extensive comparisons have been done with the solutions from Garrick[3,6] for a wide range of reduced frequencies $k(k = \omega c/2U)$. One such comparison for $k = 0.5$ is shown in Fig. 3 for one cycle of the motion with pitching $\theta = 0.1 \sin(\omega t)$, and plunging $h = 0.5c \sin(\omega t + \pi/2)$.

Note that the time step size, and hence the spacing of the vortices in the wake determined by δ, remains a free parameter in this model. Initially, a value of $\delta = \tilde{d} = 1$ was used, which gave an even spacing of vortices including the bound vortex and allowed the vortex impulse to be considered as a continuous function. However, for the more rapidly changing conditions of oscillatory motion at the higher reduced frequencies, smaller time steps were needed to have enough data points to closely follow Garrick's results. This was particularly the case for the thrust values, which vary at twice the frequency of the imposed motion (as seen in Fig. 3). Half-chord spacing, i.e., $\delta = 0.5$, was found to give acceptable thrust results for reduced frequencies up to $k = 0.5$ as used for the data of Fig. 3.

Conclusions

Considering the simplicity of the model, very acceptable results have been obtained for the normal force, pitching moment, and leading-edge suction on an unsteady airfoil. Comparisons with exact solutions for indicial and harmonically oscillating motions are excellent; the model could just as easily be applied to general nonharmonic motion as long as the motion (including time derivatives) is well defined. Continuing the studies of oscillating-wing propulsion, a general three-dimensional version of this model with a discretized spanwise vorticity distribution, including comparisons with wind-tunnel and water-channel experimental data, is currently being tested at the Institute for Aerospace Studies, University of Toronto.

Acknowledgment

This work was supported by the Natural Sciences and Engineering Research Council.

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Importance of Anisotropy on Design of Compression-Loaded Composite Corrugated Panels

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Introduction

As the use of composite materials in primary flight structures increases, so does the need to understand the effects of anisotropy on the structural response. This understanding is required in order to tailor designs that avoid the effects that degrade the structural efficiency and capitalize on those effects that improve efficiency. The study presented here investigates the importance of anisotropic terms in the design of composite corrugated panels for a range of axial compressive load intensities. Two corrugated panel configurations, namely, panels with tailored laminates and panels with a continuous laminate, were studied. These panels are of particular interest to the aircraft designer because they exhibit high structural efficiency and buckling resistance. However, because of their unique construction that allows thin laminates with few layers, they are prone to anisotropic effects and the extent to which anisotropy affects their performance is still not generally known.

Composite panels, even those constructed as balanced symmetric laminates, have anisotropic flexural stiffnesses whenever ply orientations other than 0 or 90 deg, with respect to the rectangular edges of the panel, are present in the laminate. During bending deformations of these plates, the D_{16} and D_{26} anisotropic terms cause a material-induced coupling between pure bending and twisting of the plate. In preliminary analysis of composite plates, it is common practice to ignore these anisotropic terms. Neglecting anisotropy substantially simplifies analysis and permits the analyst to use existing solutions for specially orthotropic structures and exploit symmetry in formulating the problem, thus reducing the computational cost and effort.[1]

In Ref. 1, the D_{16} and D_{26} anisotropic terms were shown to reduce the buckling resistance of balanced, symmetric, compression-loaded composite plates, and a criterion was presented for determining the conditions under which the anisotropic terms can be neglected in a buckling analysis. In the present study, the work of Ref. 1 is extended to demonstrate and determine the consequences of neglecting the D_{16} and D_{26} anisotropic terms during the optimal design of buckling critical corrugated composite panels composed of balanced, symmetric laminates.

The computer code PASCO[2,3] (panel analysis and sizing code), in which a corrugated panel is modeled as a series of linked plate elements, was used for design and analysis. PASCO allows structural analysis and optimization to be performed with or without the inclusion of the anisotropic terms. To determine the importance of the anisotropic terms, D_{16} and D_{26} were first neglected during the design analysis and then included in a final structural analysis of the optimized design. The importance of the anisotropic terms is measured by the difference between the design load and the buckling load obtained from the final structural analysis.

Design and Analyses

Of the two types of corrugated panel constructions considered in this study, the first type of construction, referred to as corrugation with tailored laminates, allows the designer to tailor the dimensions and laminate construction of the cap and

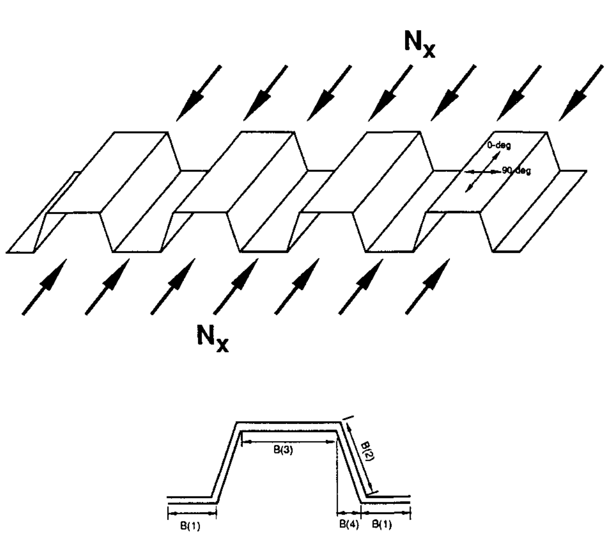

Fig. 1 General geometry of corrugated panel and repeating element.

Received May 25, 1988; revision received July 12, 1989.

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